Capturing Fuzziness and Uncertainty of Spatiotemporal Objects

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Abstract

For the majority of spatiotemporal applications, we assume that the modeled world is precise and bounded. Although this simplification is sufficient for applications like the cadastral system, it seems to be unnecessary crude for many other applications handling spatial and temporal extents, such as navigational applications. In this work, we explore fuzziness and uncertainty, which we subsume under the term *indeterminacy*, in the *spatiotemporal* context. We first show how the fundamental modeling concepts of *spatial objects*, *attributes*, *relationships*, *time points*, *time periods*, and *events* are influenced by *indeterminacy*, and show subsequently how these concepts can be combined. Next, we focus on the change of spatial objects and their geometry in time. We outline four scenarios, which identify discrete and continuous change, and we present how to model indeterminate change. We demonstrate the applicability of this proposal by describing the uncertainty related to the movement of point objects, such as the recording of the whereabouts of taxis.

1. Introduction

Spatiotemporal applications received a lot of attention over the last years from both the research and the application-oriented community. Requirements analysis [22], models [9, 30], data types [15], and data structures [28, 32, 24] are some of the main topics in this area. Although considerable research effort and valuable results *do* exist, all the studies and proposed approaches are based on the assumption that, in the spatiotemporal mini-world, objects have *crisp* boundaries, relationships among them are *precisely* defined, and *accurate* measurements of positions lead to error-free representations.

However, reality differs. Very often boundaries do not strictly separate objects but, rather, show a transition between them. Consider the example from an environmental system in which the different climate zones, such as desert and prairie, are not precisely bounded. We encounter a transition, or *fuzziness*, between them. On the other hand, in a navigational system, the position of a moving vehicle, although precise in its nature, might not be exactly known, e.g., car A is in New York. This example is characterized by *uncertainty* (i.e., *lack of knowledge* or *error*) about its actual location.

In this paper we deal with *fuzziness and uncertainty as related to spatiotemporal objects*. More specifically, we start by pointing out the semantic differences between the two cases that constitute *spatiotemporal indeterminacy*: *fuzziness,* concerning "blurry" situations, and *uncertainty,* expressing the "not-exactly-known" reality. Our goal is to clarify these terms and study their impact on the spatial and temporal domains, as well as the combined effect, spatiotemporal fuzziness and uncertainty. We show how the basic spatiotemporal modeling concepts, such as spatial objects, attributes, relationships, time points, time periods, events and change are influenced by indeterminacy. The approach presented in [13] on indeterminacy in the temporal domain, is used as a vehicle to explore fuzziness and uncertainty in spatial, temporal and spatiotemporal applications as well as to point out their differences and similarities.

The contribution of this work is twofold. First, we explore the semantics of spatial and temporal indeterminacy, to better understand their nature and behavior. For the same reason, we further focus on the differences and similarities of spatial fuzziness and uncertainty. Next, we explore the nature of spatiotemporal indeterminacy; we discuss the fundamental concept of change and the way it is affected by uncertainty and fuzziness. We give a formal way to describe it; an example demonstrates the applicability of this proposal.

To the best of our knowledge, no other work deals with indeterminate change based on the fuzzy and uncertain spatiotemporal concepts causing it. There are only few works towards spatiotemporal indeterminacy. [27] focuses on simple spatial and temporal uncertainty concepts and integrates them to describe spatial updates in a GIS database. However, the presented concepts are rather abstract and cannot immediately be applied. [23] discusses spatiotemporal indeterminacy for moving objects data. It describes an approach of how to compute and utilize error information of moving objects trajectories. The approach, however, is limited to point objects, also, it does not take temporal errors into account. [7] aims at describing the change of fuzzy features over time using a raster representation. More work exists dealing with indeterminate temporal and spatial information individually. [12] takes a probabilistic approach in handling indeterminacy of temporal information. On the other hand, research in the geography and surveying domain provides ways to describe and handle spatial indeterminacy. [8] introduces the concept of epsilon distances to quantify the cartographic error related to map production. [5] and [6] describe spatial uncertainty as related to soil boundaries. These works use fuzzy set theory for soil classification. Work on spatial indeterminacy related to resolution can be found in [34] and [35]. [33] describes of how to utilize fuzzy measures to better describe spatial relationships among determinate spatial objects. [25] and [26] take a more pragmatic approach in that the spatial world is modeled in terms of spatial data types, and fuzziness is expressed as related to the data types and the operations on them. The works on spatial indeterminacy are many and the ones presented here are only exemplary. Further readings can be found in [17].

The rest of the paper is organized as follows. Section 2 briefly presents the fundamental spatial and temporal concepts involved in the spatiotemporal application domain. Section 3 explores the semantics and gives the mathematical expression of indeterminate temporal concepts. Section 4 deals with indeterminate spatial concepts. Section 5 discusses change as the spatiotemporal concept affected by indeterminacy. This section also gives a comprehensive example for better illustration and to assess the feasibility of these concepts. Finally, Section 6 concludes with the future research plans. The Appendix shows the mathematical background used to express fuzziness and uncertainty, fuzzy set and probability theory. It further gives the differences and similarities of when applying these theories in the spatial and temporal domain.

2. Spatial and Temporal Concepts

To understand spatiotemporal indeterminacy and its concepts, it is important to realize the fundamental spatial, temporal and, by combining them, spatiotemporal concepts. Here, we give an overview of the spatial and temporal concepts that are involved in geo-referenced time-varying application environments. Later, in Sections 3, 4, and 5, we discuss how these concepts are affected by fuzziness and uncertainty and how this can be mathematically expressed.

Spatiotemporal applications can be categorized based on the type of data they manage: (a) applications dealing with *moving* objects, such as navigational; in these, objects change position in time, for example, a moving "car" on a road network, (b) applications involving objects located in space, whose characteristics, as well as their position, may *change* in time; for example, in a cadastral information system, "landparcels" change positions by changing shape, but they do not "move," and (c) applications dealing with objects which integrate the above two behaviors; for example, in environmental applications, "pollution" is measured as a *moving* phenomenon which *changes* properties and shape over time. The following modeling concepts are involved in environments like the aforementioned.

- *Spatial Objects* and their *geometry.* Objects in real world have a *position* in space. In specific application environments, the objects' position in space *matters*. These objects are called *spatial objects*, e.g., a moving "car" in a navigational system is a spatial object. Many times it is *not only the actual presence* of the object's position that matters, but its *geometry* as well. For example, while in a navigational system only the position of a car matters, indicating its actual location, in a cadastral system the *exact geometry* of a "landparcel" is of importance. The geometry of the position of a spatial object can be (of type) point, line, region or any combination thereof [19].
- *Spatial Relationships.* Spatial objects are related in space. A spatial relationship relates spatial objects, or more precise, the positions of the related objects. For example, two landparcels are neighbors, i.e., they share common borders.
- *Spatial Attributes* and their *geometry.* Objects have attributes, which characterize them. Spatial objects have, apart from descriptive attributes, also spatial attributes, e.g., the "vegetation" of a "landparcel." Values of spatial attributes depend on the referenced position and not on the object itself. If the spatial object "landparcel" changes position, then the value of "vegetation" will also change.

Spatial attributes are also related to geometries in space, as they split space in parts in whose extents the values of the spatial attributes remain the same; each part of space has (like, the objects' positions) geometry (of type) point, line, region or any combination thereof. For example, the attribute "vegetation" creates partitions of space with constant vegetation values in each partition, such as "forest", and "bushes." There are two basic types of spatial attributes: (a) those representing functions with continuous range e.g., "temperature," or "erosion." Here the geometry of the partitions is point. (b) Those representing functions with discrete range, e.g., "vegetation" represented as set of regions. In case (a), classification techniques are used to create "zones" of average values, for example, "high temperature" or "low temperature". Figure 1 shows spatial objects, spatial attributes and their geometry.

Note that not all spatial objects have spatial attributes. That depends on the application requirements. For example, no spatial attribute is usually assigned to a moving "car", while various ones (e.g., "vegetation", "soil type") may be assigned to a "landparcel."

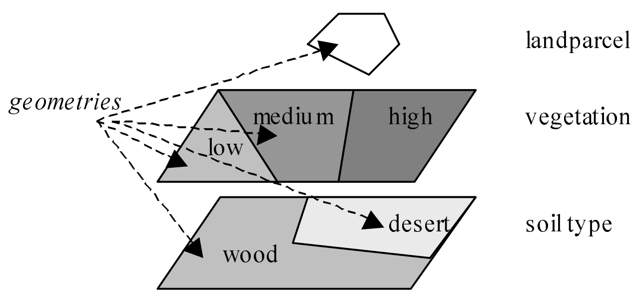


Figure 1: Spatial objects, space-depending attributes and geometries in space.

- *Time.* In literature many different models of time are presented. Some authors even propose taxonomies of time. In our work we assume a linear ordered time line, isomorphic to a finite subset of the natural numbers. The elements of this set are termed *chronons*.
- *Time points vs. Time period.* Two basic models of time are used to record facts and information of a database: time points and time period. A time point t_1 is located during a chronon, while a time period $[t_k, t_m]$, with t_k, t_m time points and $k \leq m$ has a duration and is defined as set of chronons.
- *Events* and *States.* These are two basic issues for which we want to record time. An event occurs at an exact time point, i.e., an event has no duration. An example event is a "car crash.'' A state is defined for each chronon in a time point. Hence, it has duration, e.g., a "meeting" takes place from 9 a.m. until 11a.m.

3. Temporal Indeterminacy

In temporal applications we are interested in events and their occurrence time. However, sometimes we only know approximately when an event occurred, e.g., a traffic accident happened between "2 pm and 4 pm", "on Friday", or "sometimes during the last week". The reason for this is that temporal indeterminacy has various sources [12], including dating techniques, i.e., techniques that are inherently indeterminate (e.g., Carbon-14 dating), future planning, i.e., projected completion dates are specified approximately, unknown and imprecise event times (e.g., the exact birth date of a person) and fuzzy event times, i.e., an event does not have a pronounced beginning or end (e.g., the event of Fall as judged by the changing of the weather as opposed by the date). All the examples but the fuzzy event time are characterized by a lack of knowledge and incomplete or erroneous information. In the case of a fuzzy event, the time when an event occurred cannot be stated accurately even if we would have "complete" knowledge about it.

In the following, we present models of how to represent indeterminacy in the temporal domain by adapting the model presented in [13].

3.1 Indeterminate Time Points

A time point is *determinate* if it is known when, i.e., during which chronon, it is located. Figure 2 shows the determinate point I_1, based on the approach that a chronon is longer than a time point. A time point is *indeterminate* if we do not exactly know when, but approximately during which series of chronons it is located. An indeterminate time point is described by a *lower support*, an *upper support*, and a *probability function* [12]. The *supports* are chronons that delimit the location of the time point, e.g., for time point I_2 in Figure 2, the lower support is chronon 5 and the upper support is chronon 8, whereas the probability function tells us about the likelihood where the time point is located within the range, e.g., uniform distribution tells us that it is equally likely for the time point to be located at chronons 5 to 8.

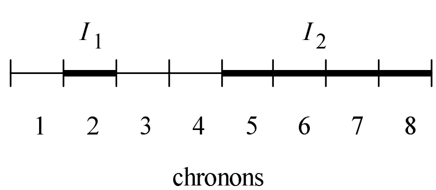


Figure 2: Determinate (I_1) and indeterminate (I_2) time points.

In the following, we use probability and fuzzy set theory to quantify indeterminacy. For a brief introduction to theses concepts refer to the Appendix. The probability mass function, p_x, for the indeterminate point x is

$$p_x(i) = P[x = i] : i \in \mathbb{N} \tag{1}$$

where $P[x = i]$ is the probability that the time point is located during chronon *i*. In our example, assuming uniform distribution, $P[I_2 = 6] = 0.25$, the probability outside the range lower support–upper support is 0. Also, all indeterminate time points are considered to be independent, i.e.,

$$P[x = i \wedge y = j] = P[x = i] \times P[y = j] \tag{2}$$

In Appendix A.3, we state that all probability distributions are fuzzy sets. By using the probability mass function as basis we obtain the following membership function:

$$\mu_x(i) = \lambda p_x(i) \tag{3}$$

In the formula (3), λ is an arbitrary scale factor relating the membership grade to the probability of a point.

3.2 Indeterminate Time Periods

A time period is a subset of the time line bounded by two time points. Depending on whether the bounding points are determinate or indeterminate, we term the time period accordingly. In Figure 3(a), I_1 and I_2 denote the indeterminate start and end point of the period. Possible periods can range from chronon 1 to chronon 8 (max), but at least have to range from 3 to 6 (min).

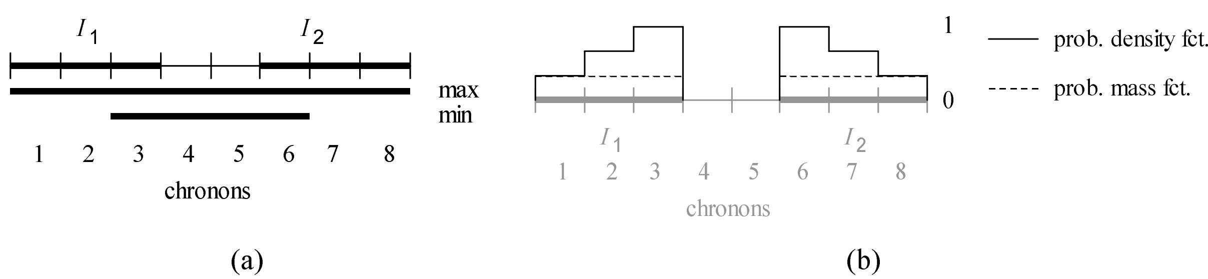


Figure 3: (a) Indeterminate time period, (b) probabilities of bounding time points.

The time period presented in Figure 3(a) can also be perceived as having a fuzzy boundary. In the following we derive a membership function, $\mu_T(x)$, returning the degree to which an arbitrary chronon *x* is part of the time period *T*. From Figure 3(a), we can deduce that chronons 4 and 5 are definitely part of the time period *T*, whereas other chronons might be. Assuming a uniform distribution of the chronons within the time points I_1 and I_2, we can see that if chronon 2 is within the period so has to be chronon 3. Further, if chronon 1 is within, so have to be chronons 2 and 3. The same is true for chronons 6, 7, and 8 of I_2. Thus, in three (all) cases chronon 3, in two cases chronon 2, and in one case chronon 1 is within period *T*. The probability mass function of I_1 and I_2 as shown in Figure 3(b) gives the probability for a chronon to be in *T*. In summing up the probability from “the outside to the inside,” we obtain a step function, the probability density function.

To derive the membership function $\mu_T(x)$ we have to split the time period T into three parts; (1) the "core" (chronons 4 and 5), (2) the intervals I_1 and I_2, and (3) the outside world. A membership grade of 1 and 0 indicate definite and no membership in the time period, respectively. All chronons in the core have a grade of 1. The grade of the chronons in the intervals is equal to the value of the probability density function. The following formula summarizes the membership function.

$$\mu_T(x) = \begin{cases} 1 & y \text{ in core} \\ \sum p(x) & y \in I_1 \vee I_2 \\ 0 & \text{otherwise} \end{cases}$$

For the case of arbitrary small chronons, the probability density function for a given subset $A \subseteq X$ is computed as $Q(A) = \int_A p(x)\, dx$.

4. Spatial Indeterminacy

The nature of spatial indeterminacy is an attractive issue in geography and spatial information science. [18] states that fuzziness is a property of a geographic entity. Furthermore, fuzziness concerns objects that cannot be precisely defined otherwise [16]. On the other hand, uncertainty results from limitations of the observation, i.e., the measurement process [18].

4.1 Indeterminate Spatial Objects, Relationships and Attributes

In this section, we point out the differences between spatial fuzziness and spatial uncertainty more prominently. Consider the example of the different climate zones, e.g., desert and prairie. Each zone is not precisely bound, but, rather, a *blurry* situation exists around their common boundaries. We can identify a location for which we are sure it is within the desert or the prairie, as well as we can find a location which is in-between. Consequently, the boundary between the two soil zones is *fuzzy*. However, for a forest divided into separate landparcels, we can clearly say what tree belongs to what landparcel. The boundaries between the land parcels are *crisp* and thus, *certain*.

In contrast, let us consider the position of a moving vehicle whose location is not exactly known, e.g., a car is in New York. This example is characterized by a *lack of knowledge* about the car's location. The fact that the car is somewhere, is precise. However the lack of knowledge we have about its position introduces *uncertainty*. Without further knowledge, we can only give the probable area the car is in.

These examples indicate that the distinguishing element between fuzzy and non-fuzzy facts is a *crisp boundary*, i.e., we cannot clearly say what belongs to what. The concept of boundary introduces the *interior/exterior* notion, i.e., what is within the boundary and what is outside. Spatial fuzziness occurs (a) in the relationships among spatial objects and (b) in spatial attributes.

On the other hand, the distinguishing element between uncertain and certain facts is *the lack of, or the error in our knowledge*, i.e., we do not have sufficient knowledge about an otherwise precise fact. As a result, spatial uncertainty can refer to the degree of knowledge we have about an object's position. Uncertainty about an object's position leads to uncertainty about the spatial relationship among this object and its neighbors, e.g., if the exact boundary of a land parcel is not known, then, the exact spatial relationships with its neighboring land parcels are not known either. Furthermore, uncertainty can exist for spatial attributes, when knowledge about them is limited. Table 1 summarizes these results.

Spatial Concepts/Indeterminacy	Fuzziness	Uncertainty
Objects' position	–	√
Relationship among objects	√	√
Spatial attribute	√	√

Table 1: Spatial concepts and indeterminacy.

4.2 Indeterminate Geometry

In this section, we define more closely what indeterminacy means in relation to geometry. Geometry is essential in defining the concepts of spatial object and spatial attribute. Further, spatial relationships are defined in terms of the positions and thus the geometry of spatial objects.

Points and *regions* are the most commonly met simple geometries in spatial applications, while *line*, the third popular geometry, can be regarded as a special case of a region. For the rest of the paper, we only consider cases of simple geometries, i.e., points and regions with no holes and no disconnected parts. The goal is to examine in what ways fuzziness and uncertainty affect these geometries.

Geometry: Point

- Uncertain point: a point can be crisp and uncertain, e.g., we know the approximate position of a car and can give probabilities for its location.
- Fuzzy point: not applicable, since the concepts of boundary and interior/exterior do not exist here.

Geometry: Region

- Uncertain region: consider the example of a landparcel with "not-exactly-known" (missing data) boundaries.
- Fuzzy region: since a region is determined by its boundaries (something is inside/outside, or left/right), a region can be fuzzy, e.g., consider climate zones, whose boundaries are not crisp, but transitional.

4.2.1 Indeterminate Points

Not knowing the position of a car is connected to the lack of information about its position. Nevertheless, we still need to describe and to represent its position with any degree of information available.

We conceive *Space* as a set of points, homeomorphic to $\mathbb{N}^2$, the exact position of an object with geometry point is determinate, if it can be mapped onto a single point $p \in \mathbb{N}^2$. The position is indeterminate if it can only be mapped to a set of points, i.e., the exact position is unknown. A probability function describes the likelihood for each point to be the position, e.g., uniform distribution tells us that there is an equal chance for each point. The probability mass function, p_x, for the indeterminate point *x* is

$$p_x(i) = P[x = i] : i \in \{\mathbb{N} \times \mathbb{N}\} \tag{4}$$

where $P[x = i]$ is the probability that the position is mapped to point *i*, with *i* being a Cartesian coordinate.

As in the case of the time period, the probability that the position is outside the point set is 0. Further, all indeterminate positions are considered to be independent (cf. Section 3.1).

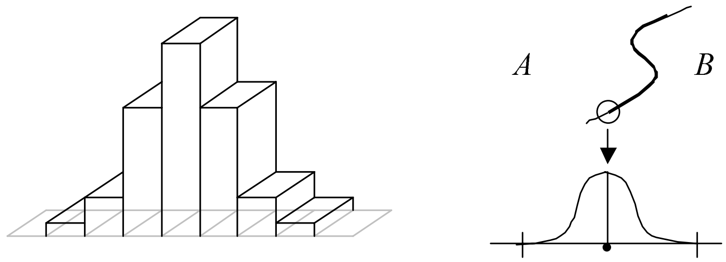
What applies to time points, (cf. Section 3.1), can also be applied to indeterminate points in the spatial context; probability distributions describing positional indeterminacy can always be interpreted as fuzziness.

4.2.2 Indeterminate Regions

The mathematical notation of indeterminate points can be extended to cover indeterminate regions as well. Indeterminate regions comprise uncertain and fuzzy regions.

A *region* is a part of space bounded by a connected set of points, the boundary. A region can be determinate if the boundary points are determinate. Consequently, indeterminate points bound an indeterminate area. This definition is analogous to Section 3.2, which presented the concept of an indeterminate time period. The following example illustrates this point.

Uncertain Regions. Consider a map made up of two discrete regions, A and B, sharing a common boundary. If we repeatedly digitize the map and thus the boundary, assuming that our process introduces errors, we obtain a set of points that lie more or less close to the actual boundary line. However, there will be more points closer to the actual location of the line than further away from it. Due to lack of better knowledge, this distribution might take the form of a normal distribution whose mean is centered at the "true" location of the line. In Figure 4(a), we show the normal distribution of a particular boundary point. In the continuous case, the probability function will look like in Figure 4(b).



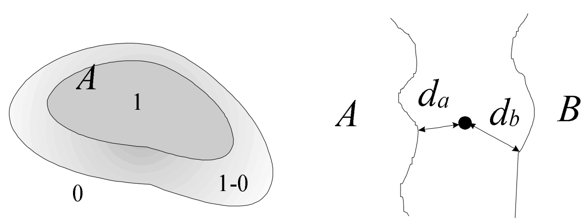
(a) (b)

Figure 4: Boundary point probability.

Analogously, we can describe this uncertain region using a membership function. To determine this function that returns the grade to which an arbitrary point in space belongs to an area, we use a similar approach as in the case of the time period (cf. Section 3.2). We split the underlying space into three parts, (i) the core of the area, (ii) the boundary region, and (iii) the outside. Consequently, a membership function for area *A* can be specified as follows.

$$\mu_A(i) = \begin{cases} 1 & i \in A \wedge i \notin B \\ \sum p(x) & i \in A \wedge B \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

In the above formula, area *B* stands for the outside of area *A* and $p(x)$ is the probability mass function of a point for being in area *A*. The argument of the membership function is a point and it returns a grade for the membership of this point in area *A*. The grade is 1 if the point is a definite member of the area and 0 if it is definitely not a member of the area. Otherwise the grade is between 1 and 0 (cf. Figure 5(a)).



(a) (b)

Figure 5: Boundary point probability.

Fuzzy Regions. In the above approach, we use the positional probability function of the boundary points to devise a membership function. However, this approach is only feasible in case the probability function is *known* and *simple*, i.e., there is one probability function describing the distribution of all points in the boundary. If there were many probability functions, in the worst case a different one for each boundary point, the membership function would become too complex to be useful. On the other hand, in some cases, we do not have "any information at all" about the boundary of a region. Consider here, the transition between soil zones as described in Section 4.1. The boundary exists because of the very nature of a phenomenon is not crisp and, thus, to give a probability function describing it is not possible, or does not make sense. This illustrates the critical case for which fuzziness relieves uncertainty. We can still derive a valid membership function in *assuming a smooth and steady transition* from one zone to the other. A membership function for soil zones, as shown in Figure 5(b), could be characterized by the following formula (cf. [26]),

$$\mu_A(x,y) = \begin{cases} 1 & \text{if } (x,y) \in A \\ 1 - \dfrac{d_a}{d_a + d_b} & \text{if } (x,y) \notin A \wedge \ (x,y) \notin B \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where d_a and d_b are the distances from a point (x,y) to the core area of the soil zones *A* and *B*.

A formula for a distance *d* from an arbitrary point given by its coordinates (x,y) to an area *A* with the boundary B_A is as follows

$$d\big((x,y),B_A\big) = \min\big\{\text{dist}\big((x,y),(m,n)\big) \mid (m,n) \in B_A\big\} \tag{7}$$

where dist(*p*,*q*) is the Euclidean distance between two points $p,q \in \mathbb{R}^2$.

The underlying assumption above is that the transition between the climate zones is linear. However, the effect of other transitions on the membership function would be a change of the formula describing the membership grade for positions outside the core.

Other examples in this case are the boundary problem as experienced in the context of soil profiles, soil maps, and land evaluation classification [5, 6].

5. Spatiotemporal Indeterminacy

After showing the nature of spatial and temporal indeterminacy as well as the way to model it, we aim for describing the combined phenomenon, *spatiotemporal indeterminacy*. We first give some examples and subsequently present the concept that is fundamental to spatiotemporal scenarios, namely *change*. We show how to model *indeterminate change* using probabilities and fuzzy set theory.

Consider a moving object whose position is sampled in time, i.e., moving vehicles, persons, military units, etc. It is reasonable, for the purposes of the example to assume that the extents of these objects do not matter in a given application context, and, thus, can be reduced to points. In order to record the movement of a moving object, we would have to know its position at all times, i.e., on a continuous basis. However, GPS and telecommunications technologies only allow us to sample an object's position, i.e., to obtain the position at discrete instances of time, such as every few seconds. A first approach to represent the movements of such an object would be to store the position samples. This would imply that we could not answer queries about the object's movements at times in-between sampled positions. Rather, to obtain the entire movement, we have to interpolate the positions. The simplest approach is to use linear interpolation, as opposed to other methods such as polynomial splines [2].

For areal objects the change of position includes the change of their centroid and the change of their shape, which has to be interpolated as well. Consider here the example of a coastline that bounds a landmass, e.g., an island. Two processes influencing the coastline make an island an indeterminate region. The tides have (i) a short-term effect, whereas (ii) over a longer period of time a general drift affects the shoreline as well. If one is only interested in the general drift, the tidal effect can be modeled as a fuzzy boundary that changes with time (general drift).

Spatiotemporal indeterminacy can have more than one source, i.e., it can be the result of the combined effects of temporal and spatial indeterminacy. In the following we will show possible scenarios on the context of spatiotemporal data.

5.1 Spatiotemporal Scenarios and Indeterminate Change

As in spatiotemporal applications we are interested in spatial objects, relationships and attributes over time, we, in reality, do record their evolution, or *change* in time. Thus, change is the most important concept in the spatiotemporal context, and will in the following serve as the basis to evaluate spatiotemporal indeterminacy. As stated in literature [9, 15, 22] change (i) can either occur on a discrete or on a continuous basis and (ii) can be recorded in time points or in time periods.

Table 2 illustrates the fours *change* scenarios we encounter in the spatiotemporal context by using a 3-dimensional representation of the temporal change of geometry. Space (*x*- and *y*-coordinates in the horizontal plane) and time (time-coordinate in the vertical direction) are combined to form a three dimensional coordinate system. In the change scenarios, the elements that can be indeterminate (with respect to an object) are *geometry*, *time point,* and *time interval*. We use a point geometry to keep the illustrations simple. However, the same four change scenarios apply to other geometries. To distinguish discrete from continuous changes, a discrete change of geometry from G_i to G_{i+1} is indicated by using an arrow in the spatial plane as opposed to a line in case of a continuous change. In the following, we examine each of them with respect to indeterminacy.

The first case, Scenario 1 in Table 2, is the *discrete change of a geometry recorded in time points*. Here, geometry stays constant for some time and then changes instantly. The geometry is sampled at constant time intervals *dt*. The geometry and/or the time point can be indeterminate.

The second case, Scenario 2 in Table 2, is the *continuous change of a geometry recorded in time points.* Here, we sample a constantly changing geometry at time intervals *dt*. Knowing a geometry only

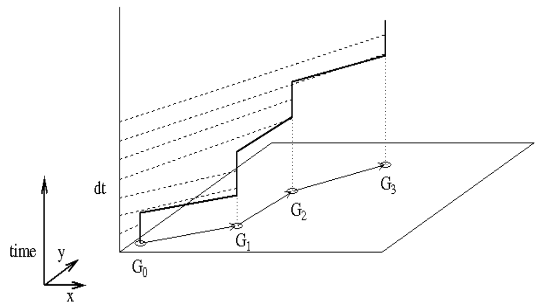
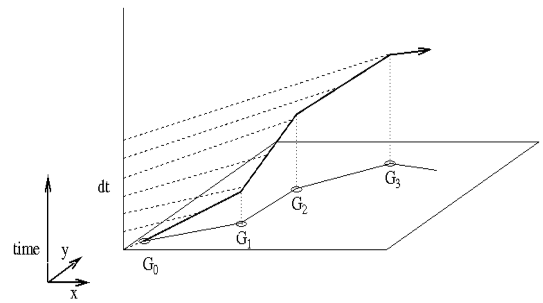
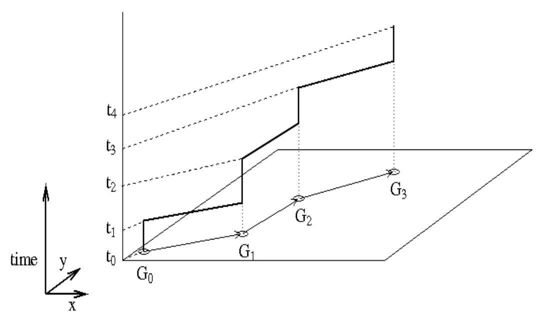
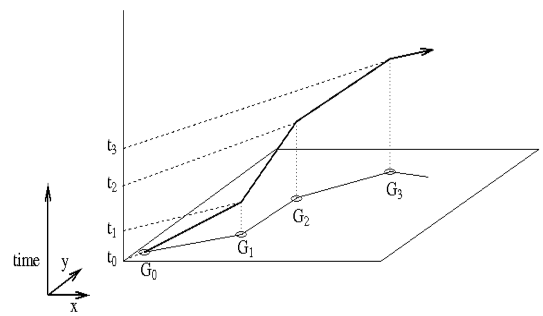
Change/ Time	Discrete	Continuous
Point	**1)** A geometry is recorded at a time point. The geometry may or may not differ from the previously recorded one. We do not know when the change occurred. 	**2)** A geometry is sampled at *time points*. In between time points we have no knowledge about the geometry. 
Period	**3)** A geometry is valid for a given *time period*. After a change, a new time period starts. 	**4)** A geometry is sampled at time points, the starting and end points of the time period. Further, a time period is assigned a *"change" function* that models the positional change within the time period. 

Table 2: Four change spatiotemporal scenarios.

at time points has two implications, (i) recording geometries at time points means assessing a momentary situation without inferring anything about the geometry prior or past the time point. Consequently, (ii) time and space are independent; not knowing the exact extent of the geometry does not affect the time interval and vice versa.

In contrast, Scenarios 3 and 4 in Table 2, suggest that a *change function* of the form $C: t_x \rightarrow G_x$ exists that determines a geometry G_x for a time point t_x in an interval spatially bounded by the two geometries G_i and G_{i+1} and temporally bound by the time interval $T_i = [t_i, t_{i+1}]$. The change function C can be different for every time interval.

The third case, Scenario 3 in Table 2, is the *discrete change of a geometry recorded in time intervals.* The objective is to "begin" a new time interval when a spatial change occurs, i.e., a new time intervals start at the time points t_0 through t_4. The geometry is constant within a time interval. In this case, spatial and temporal indeterminacy affect each other. Dealing with indeterminate spatial extents, e.g., uncertainty induced by measurement errors, implies that the time point at which a change occurs cannot be detected precisely. On the other hand, having an indeterminate temporal event, e.g., clock errors, introduces spatial indeterminacy.

The last and most complex case, Scenario 4 in Table 2, is the *continuous change of a geometry recorded in time intervals.* This case is based on the fact that for a given time interval $T_i = [t_i, t_{i+1}]$, there exists a change function that models the transformation from geometry G_i to G_{i+1}. Each of these factors, i.e., (i) the time interval, (ii) the geometry, and (iii) the change function, can be subject to indeterminacy.

In the simplest case, the geometry G_i and G_{i+1} and the time interval T_i are *determinate*, and the change function returns a determine geometry G_x for a given time point $t_x \in T_i$. Here, we assume that the change function returns the geometry coinciding with the actual movement. Is this not the case, the change function *interpolates* in between the geometries G_i to G_{i+1} and returns indeterminate geometry. An example is to use linear interpolation, i.e., the two geometries G_i to G_{i+1} are considered to be the endpoints of a line. Section 5.2 gives an elaborate example of a change function for this case.

Geometry (G_i, G_{i+1})	**Time (t_i, t_{i+1})**	**Change**
Determinate	Determinate	$C: t_x \rightarrow G_x$, where G_x, depending on the change function, is determinate or indeterminate ($\tilde{G}_x$)
Indeterminate	Determinate	(a) $C: t_x \rightarrow \tilde{G}_x$, where $\tilde{G}_x$ represents either a probability function, $P_x(i)$, or a membership function, $\mu_x(i)$ (b) $\mu_x(i,t)$ or $P_x(i,t)$

Table 3: Change scenarios without temporal indeterminacy.

If we further allow G_i and G_{i+1} to be *indeterminate*, our change function would in any case return an indeterminate G_x. In the following, we use the "~" symbol on top of the parameter to denote indeterminacy. This means that if geometry is described by a probability or membership function, this very function is subject to change in the time interval T_i.

Following the idea from before, we would have a change function that returns a probability or membership function for a given t_x (cf. Table 3(a)). However, by integrating the temporal component, we obtain a spatiotemporal probability or membership function, i.e., a function that changes with time (cf. Table 3(b)).

Up until now, we always considered time to be determinate. We use time points to determine the start and the end of the current time interval T_i, and to denote the time point in question, t_x. In case t_i and t_{i+1} are indeterminate, we cannot state the beginning and the end of the time interval precisely. Thus, the association of a geometry (indeterminate or not) to a time point becomes indeterminate. However, this affects mainly the change function and can be considered in adapting its form. In considering an indeterminate time interval, we cannot, for any time point in the time interval, give a geometry as it would be unaffected by determinate time, but the indeterminate time contributes some additional indeterminacy. Table 4 adapts the approach shown in Table 3 to accommodate temporal indeterminacy.

Geometry (G_i, G_{i+1})	Time (t_i, t_{i+1})	Change
Determinate	Indeterminate	$C:\tilde{t}_x \rightarrow \tilde{G}_x$
Indeterminate	Indeterminate	(c) $C:\tilde{t}_x \rightarrow \tilde{G}_x$, where $\tilde{G}_x$ is either a probability function, $P_x(i)$, or a membership function, $\mu_x(i)$ (d) $\mu_x(i,\tilde{t})$ or $P_x(i,\tilde{t})$

Table 4: Change scenarios incorporating temporal indeterminacy.

The central element of spatiotemporal indeterminacy is the change function manipulating geometries. Since the geometry of a position can be of type point, line, or region, the change function can be seen similar to a morphing algorithm between different instances of geometries. The following section illustrates this approach by giving an example. We describe of how to represent the movement of vehicles and how to quantify the error associated with such a representation.

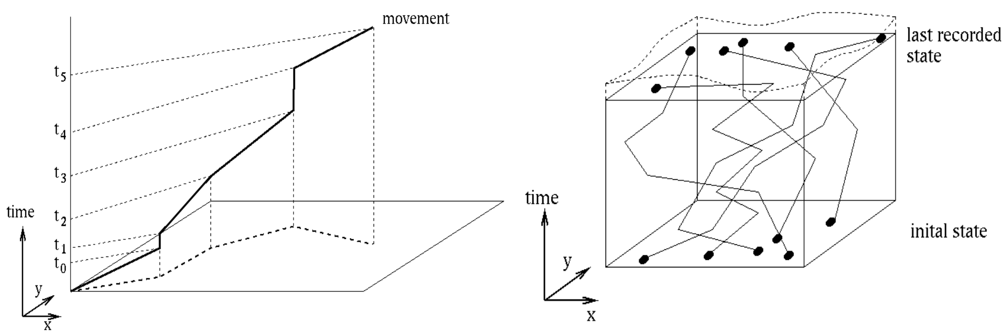
5.2 An Example of Use – Tracking Vehicles

As mentioned, we can identify areal objects whose extents do not matter in a given application context, and, thus, can be reduced to points. Consider, an application scenario in which we track the continuous movement of taxis equipped with GPS devices that transmit their positions to a central computer using either radio communication links or cellular phones. At the central site, the data is processed and utilized.

5.2.1 Acquiring Movement – Sampling Moving Objects

To record the movement of an object, we would have to know the position at all times, i.e., on a continuous basis. However GPS and telecommunications technologies only allows us to sample an object's position, i.e., to obtain the position at discrete instances of time such as every few seconds.

The solid line in Figure 6(a) represents the movement of a point object. Space (x- and y-axes) and time (t-axis) are combined to form one coordinate system. The dashed line shows the projection of the movement onto two-dimensional space (x and y coordinates). A first approach to represent the movements of objects would be to store the position samples. This would mean that we could not answer queries about the objects' movements at times in-between sampled positions. Rather, to obtain the entire movement we have to *interpolate*. The simplest approach is to use linear interpolation, as opposed to other methods such as polynomial splines [2]. The sampled positions become the end points of line segments of polylines. The movement of an object is represented by an entire polyline in three-dimensional space. In geometrical terms, the movement of an object is termed a *trajectory* (we will use "movement" and "trajectory" interchangeably). Figure 6(b) shows the spatiotemporal space (the cube in solid lines) and several trajectories (the solid lines). The top of the cube represents the time of the most recent position sample. The wavy-dotted lines at the top symbolize the growth of the cube with time.



(a) (b)

Figure 6: Movements and space.

5.2.2 Measurement Error

An error can be introduced by inaccurate measurements. Using GPS measurements in sampling, the error can be described by a probability function, a bivariate normal distribution P_1, also visualized in Figure 7.

$$P_1(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (8)$$

A typical GPS module used in vehicle navigation systems is the CrossCheck AMPS Cellular from Trimble Navigation Ltd., which has an error of $2m$ (standard deviation σ) [29]. For more details on this error measure refer to [23].

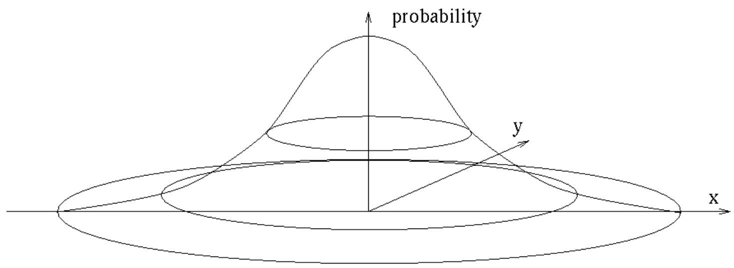


Figure 7: Positional error in the GPS.

5.2.3 Which Scenario?

In the schema of Section 5.1, the sampling approach to assess the movement of objects is characterized by scenario 4. Table 3 and Table 4 establish a foundation for giving a change function in between sampled position. Table 4 gives function templates in case the times of sampling are not known precisely. However, GPS allows for precise timing and, thus, we neglect the effect of time. In Table 3, Scenario 1 (determinate geometry) gives a function template in case the sampled positions are known precisely. As we just saw, GPS measurements are accurate but not precise. Thus, Scenario 2 (indeterminate geometry) seems to be a match for our problem. The following section shows how to establish a change function to determine the position of the moving object in between sampling. We initially assume precise position samples.

5.2.4 Sampling Uncertainty

As mentioned, we capture the movement of an object by sampling its position using a GPS receiver at regular time intervals. This introduces *uncertainty about the position* of the object in-between the measurements. In this section, we give a model for the uncertainty introduced by the sampling, based on the sampling rate and the maximum speed of the object.

The uncertainty of the representation of an object's movement is affected by the frequency with which position samples are taken, the *sampling rate*. This, in turn, may be set by considering the speed of the object and the desired maximum distance between consecutive samples. Let us consider the running example, in which we want to record the movements of taxis.

Example. As a requirement to the application, the distance between two consecutive samples should be maximally $10m$. If the maximum speed of a taxi is $150km/h$, this means that we would need to sample the position at least 4.2 times per second. If a taxi moves slower than its maximum speed, the distance between samples is less than $10m$. How do the position samples resemble the true movement of the taxi? Consider the three trajectories shown in Figure 8. Each is possible given the three measured positions P_1 through P_3. However, by just "looking" at the three positions, one would assume that the straight line best resembles the actual trajectory of the taxi.●

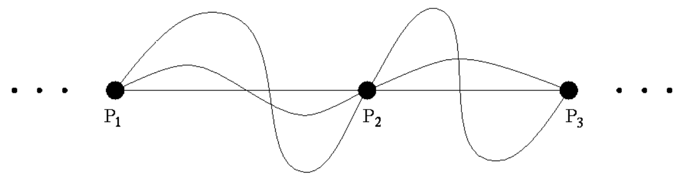


Figure 8: Possible trajectories of a moving object.

Since we did not have positional measures in-between position samples, the best we can do is to *limit the possibilities of where the moving object could have been*. We constrain the trajectory of the object by what we know about the object's actual movement. Considering the trajectory in a time interval [t_1, t_2], delimited by consecutive samples, we know two positions, P_1 and P_2, as well as the object's maximum speed, v_m (cf. Figure 9). If the object moves at maximum speed v_m from P_1 and its trajectory is a straight line, its position at time t_x will be on a circle of radius $r_1 = v_m(t_1 + t_x)$ around P_1 (the smaller dotted circle in Figure 9). Thus, the points on the circle represent the furthest away from P_1 the object can gotten at time t_x. If the object's speed is lower than v_m, or its trajectory is not a straight line, the object's position at time t_x will be somewhere within the area bounded by the circle of radius r_1. Similar assumptions can be made on the position of the moving object with respect to P_2 and t_2 to obtain a second circle of radius r_2. The constraints on the position of the moving object mean that the object can be anywhere within the intersection of the two circular areas at time t_x. This intersection is shown by the shaded area in Figure 9. We use the term *lens* for this area of intersection. Since we do not have any further information, we assume a uniform distribution for the position within the lens, i.e., the object is equally likely anywhere within this lens shape.

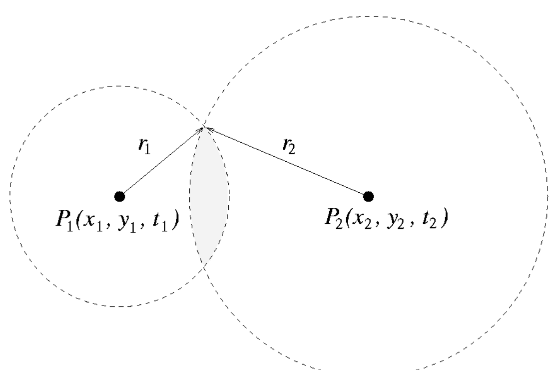


Figure 9: Uncertainty between samples.

Thus, the sampling error at time t_x for a particular position can be described by the probability function shown in Equation (9), where r_1 and r_2 are the two radii described above, s is the distance between the measured positions P_1 and P_2, and A denotes the area of the intersection of the two circles.

$$P_2(x,y) = \begin{cases} \frac{1}{A} & \text{for } x^2 + y^2 \le r_1^2 \wedge (x-s)^2 + y^2 \le r_2^2 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

To eliminate the radii in favor of the max speed and times, we can substitute $v_m(t_1 + t_x)$ and $v_m(t_2 - t_x)$ for the r_1 and r_2, respectively. This function describes the position of the moving object in between position samples. Thus, this function is an instance of the function template as described in Scenario 1 of Table 3.

5.2.5 Combination of Error Sources – a Global Change Function

Table 3 gives a framework change function that incorporates indeterminate positions. In the context of this example, this translates into adapting Equation (9) such that the values for x and y are not precise but affected by the measurement error as described in Section 5.2.2. Although it seems trivial at first, this requires some heavy mathematical manipulation that would be beyond the scope of this work.

A general mathematical framework suitable for this problem is *Kalman filtering* [20], which is a method to combine various error prone measurements about the same fact into a single measurement with a smaller error. This mathematical framework stipulates a method to combine uncertainty to reduce the overall error. Assuming the measurements refer to position samples of a continuous movement in time, we can use *Kalman smoothing* to determine the positions at times that are in between the measured ones [1]. Examples of applying Kalman filtering to the domain of vehicle navigation are the integration of three independent positioning systems such as dead reckoning, map matching, and GPS, to determine the precise position of vehicles [21]. Similar formal frameworks are used in commercially available car navigation systems [3].

6. Conclusions and Future Work

The work presented in this paper concerns the spatial, temporal, and spatiotemporal indeterminacy, i.e., fuzzy and uncertain phenomena. We first show how the fundamental modeling concepts of *spatial objects*, *attributes*, *relationships*, *time points*, *time periods*, and *events* are influenced by *indeterminacy*, and how we can combine them. Next, we focus on the change of spatial objects and their geometry in time. We argue that change can occur on a discrete and on a continuous basis, as well as it can be recorded in time points and time periods. By combining these concepts we present with four different change scenarios, which are affected by indeterminacy to a various degree. The indeterminacy of change is formalized and combines the spatial and temporal concepts. Finally, the rather general concepts are applied to existing application areas. We discuss uncertainty existing in the context of moving-point-object applications. We give a change function to describe the position of moving objects in over time based on positional samples. The change function is influenced by measurement errors and sampling uncertainty.

Although mentioned, the paper does not discuss, directly, indeterminacy as related to relationships among spatial, temporal, or spatiotemporal objects. An extension of this work towards this direction is essential. Also, the mathematical models we presented are concrete enough to describe and motivate indeterminacy related to the temporal, spatial, and spatiotemporal domain. However, to actually implement these concepts, more detailed mathematical formulas are needed. Finally, in a more general framework, this work points towards the development of spatiotemporal data types and data structures incorporating indeterminacy.

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Appendix A - An Introduction to Indeterminacy Measures

This appendix gives some mathematical background on *fuzzy set theory* and *probability theory* relevant for Sections 3, 4, and 5. Sections A.4 and A.5 show the differences of these two concepts when they are applied in a temporal and spatial context.

A.1. Fuzzy Set Theory

Fuzzy set theory [36] is an extension and generalization to Boolean set theory. Let X be a classical (crisp) set of objects, called the universe. Membership in a classical subset A of X can be described by the characteristic function $\chi_A : X \to \{0,1\}$ such that for all $x \in X$ the following holds.

$$\chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

This function discriminates sharply between the members and non-members of the set A. We can generalize this function by mapping the elements of set X not to the set $\{0,1\}$ but rather to the real interval $[0,1]$. Now, elements have no strict membership, but rather have a *degree of membership* in the set in question. Larger values indicate higher grades of membership.

Let X be the universe, the *membership function*

$$\mu_{\tilde{A}} : X \to [0,1]$$

returns for a given *element* of X the degree it belongs to the *fuzzy set*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}.$$

All elements of X are evaluated towards a membership in $\tilde{A}$. Those elements that do "not at all" belong to the set have a degree of membership $\mu_{\tilde{A}}(x) = 0$, whereas the elements that "totally" belong to the set have a value of $\mu_{\tilde{A}}(x) = 1$. Although fuzzy set theory seems sound and simple, to actually apply it is difficult. The problem arises from the choosing an appropriate membership function.

A.2. Probability Theory

Let again X be the universe, then the probability measure P is a mapping $2^X \to [0,1]$ that assigns a number $P(A)$ to each *subset* of X, and satisfies the Kolmogorov axioms (cf. [11]):

$$P(X) = 1; P(\varnothing) = 0$$
$$P(A \cup B) = P(A) + P(B); \quad \textit{iff} \ A \cap B = \varnothing;$$

$P(A)$ is the probability that an ill-known single-valued variable y ranging on X hits the fixed well-known set A. Given the case the underlying domain of the universe X is discrete, the *probability mass* *function* $p(x) = P(\{x\})$ returns the probability for the single element $x \in X$. The probability density function returns the probability $Q(A) = \int_A p(x)dx$ for a given subset $A \in X$.

A.3. General Differences between Fuzzy Set and Probability Theory

Very often fuzzy values are misunderstood to be probabilities, or fuzzy logic is misunderstood as a new way to handle probabilities. This is not true, since a minimum requirement of probabilities is additivity, i.e., all probabilities for alternative events have to sum up to one. This is not the case for membership grades. In mathematical terms, the membership function $\mu_{\tilde{A}}(x)$ is similar to $P(\{x\}) = p(x)$, except for the above condition, $\sum_{x \in X} p(x) = 1$, must hold while this is not true for $\mu_{\tilde{A}}$ (cf. [11]).

Also, a membership grade is defined only for one element of a set and not for a subset. Probabilities can be given for any subset, i.e., also one element. However, all probability distributions are fuzzy sets. As fuzzy sets and logic generalize Boolean sets and logic, they also generalize probabilities.

A.4. Differences between Fuzzy Set and Probability Theory in the Temporal Domain

The phenomenon of whether a point is inside or outside the time period *cannot be described by using probabilities*. The reason is the additivity criterion described in Section A.2. This is violated since all membership grades do not add up to 1. Consider here the chronons in the time period that have a membership grade of 1.

If we revisit the indeterminate time point (cf. Figure 2), we can see that if we want to describe the membership grade of a chronon for "belonging" to a particular point, with 1 representing a certain membership, each of the chronons would have a membership grade of 1/3, which is equal to the probability that each of the chronons is the actual time point. Since the membership grade is equal to the probability, they add up to 1. However, in the case of a time period bounded by indeterminate time points, we have regions that have a membership grade of 1 as well, i.e., they do not add up to 1. In this case, *the membership function is not the "same" as the probability function, but the latter is used to derive the former*.

A.5. Differences between Fuzzy Set Theory and Probability Theory in the Spatial Domain

Probability functions are used to describe uncertain positions. Consider here an unknown location, whose positional probability is scattered over a region, i.e., a set of points. If we state that the location is at one of the points of the region, then this statement is true, since all the probabilities scattered over the region add up to one. In other words probability can also be defined for a set of points as opposed to only one point (cf. Section A.3). On the other hand, if we, instead of giving a probability for a point, devise a membership grade, the sum of all membership grades over all the points in the region has no particular meaning. It is merely an arbitrary number. The membership grade of a point tells us about the belief that a point belongs to a particular set, e.g., the soil type desert. In terms of boundary, it does not tell us which points in the "fuzzy" region are the most likely ones. Thus, fuzzy concepts are related to *what belongs to what extent to a given set*, i.e., what is "in" and what is "out." Probabilistic concepts are related to what is the most likely position, i.e., where is the border for what is in and what is out. Fuzzy concepts refer to relative aspects whereas probabilistic concepts refer to absolute aspects of a spatial scenario.